

Optimal minimum wages in spatial economies

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Reading

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Motivation

- **Minimum wages (MWs) are a popular labour market policy**
 - Empirically controversial (Neumark and Shirley, 2021)
 - Theoretically consensual (Manning, 2021)
- **Spatial implications of minimum wages (MW) not well understood**
 - Impact depends on *regional* productivity \Rightarrow Impact on spatial inequality?
 - Role of goods and factor mobility for aggregate and distributive effects?
- **What is the optimal MW?**
 - Many advocates for ambitious MWs in the range of 60-70% of median wage

We develop a quantitative spatial general equilibrium model to offer answers

What we do

- **New quantitative spatial GE model**

- *Canonical*: Mobility, costly commuting, trade, housing (e.g. Monte et al., 2018)
- *“New”*: Heterogeneous firms & wage distribution by region; potential for monopsony power (upward-sloping labour supply); elastic aggregate labour supply

- **Data and quantification for 4,421 micro regions**

- 30M workers from matched employer-employee data (IAB, BeH & IEB)
- 20M residential obs. 2007-2018: property price index (Ahlfeldt et al., 2022)

- **Quantitative analysis**

- Reduced-form evaluation of 2015 German federal minimum wage
- Structural evaluation of 2015 German minimum wage & **range of alternatives**

What we contribute & find

- ① **Theoretical:** Employment changes hump-shaped in regional productivity
- ② **Empirical:** Reduced-form evidence for hump shape \Rightarrow monopsony
- ③ **Methodological:** New QSGE model with monopsonistic labour markets
- ④ **Quantitative:** Evaluate aggregate effects of German minimum wage 2015
 - welfare +2.1%, spatial equity +0.5%, FTE employment -0.3% (-100K)
- ⑤ **Normative:** Optimal minimum wage wrt (i) employment (L), (ii) welfare (V), (iii) spatial dispersion (D), (iv) wage inequality
 - *Welfare-max. federal MW:* 58% of national mean ($\Delta V +4\%$, $\Delta D -5\%$, $\Delta L -4\%$)
 - *Spatial equity-max. MW:* 43% of nat. mean ($\Delta V +0.8\%$, $\Delta D +1.5\%$, $\Delta L +0.8\%$)
 - *Emp.-max. regional MW:* 50% of local mean ($\Delta V +3.9\%$, $\Delta D 0\%$, $\Delta L +1.1\%$)

Literature (selection)

- **Employment (reduced-form):** MW may (Meer and West, 2016; Clemens and Wither, 2019) or may not have negative employment effects (Dube et al., 2010; Cengiz et al., 2019).
- **Germany:** Small employment effects, e.g. Ahlfeldt et al., 2018; Bossler and Gerner, 2019; Caliendo et al., 2018; Dustmann et al., 2022
- **Spatial:** Monras (2019) and Simon and Wilson (2021) explore MW under competitive labour markets; Bamford (2021) shows that monopsony power affects agglomeration
- **Macro/ labour:** Berger et al. (2022); Drechsel-Grau (2021); Haanwinckel (2020); Vergara (2021)

Data, background & stylized facts

Data

- **Employment, establishments and wages.** Individual-level panel data containing workplace, residence, establishment, wage, and characteristics such as age, gender, and skill ($\approx 30\text{M}$ employees).
- **Hours worked.** Imputed from auxiliary regressions that account for sector, state and socio-demographic attributes (Ahlfeldt et al., 2018). 1% sample of 2012 census. Full-time $\approx 40\text{hrs}$, part-time $\approx 21\text{hrs}$, marginal $\approx 10\text{hrs}$.
- **Real estate.** Area-year housing cost index based on $\approx 20\text{M}$ residential observations btw 2007-2018 (Ahlfeldt et al., 2022).
- **Trade.** Bilateral trade volumes between German counties in the year 2010 for different product groups (Forecast of Nationwide Transport Relations).
- **Spatial unit.** 4,421 municipalities (Verbandsgemeinden) as defined in 2018.

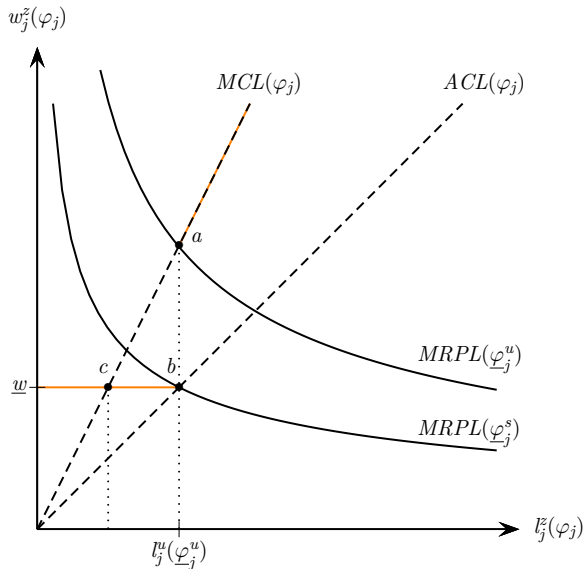
Background

- Set at €8.50 per hour (48% of full-time mean wage) in 2015, virtually universally binding.
- Raised to €8.84 (2017), €9.19 (2019), €9.35 (2020), €9.50 (Jan 2021), €9.60 (Jul 2021), €9.82 (Jan 2022), €10.45 (Jul 2022), €12.00 (Oct 2022).
- **MW fluctuated between 47-49% of the national mean wage \Rightarrow Introduction of minimum wage treated as a singular intervention in 2015.**
- **Stylized evidence points to spatially imperfect labour market model.**

Stylized facts

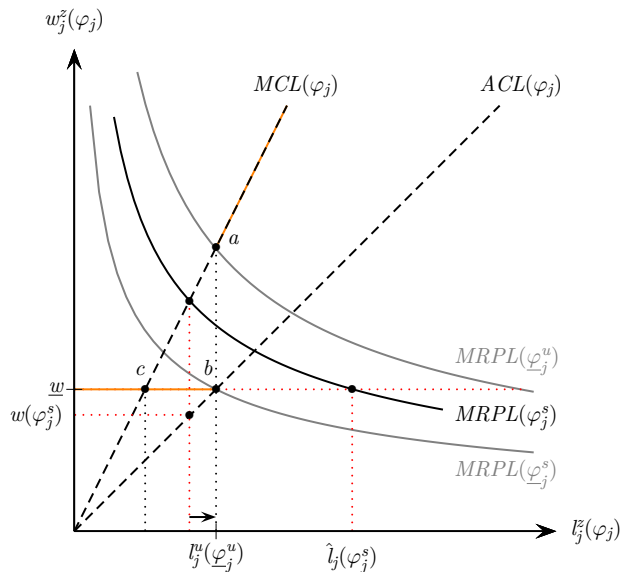
Partial equilibrium analysis

Optimal firm employment



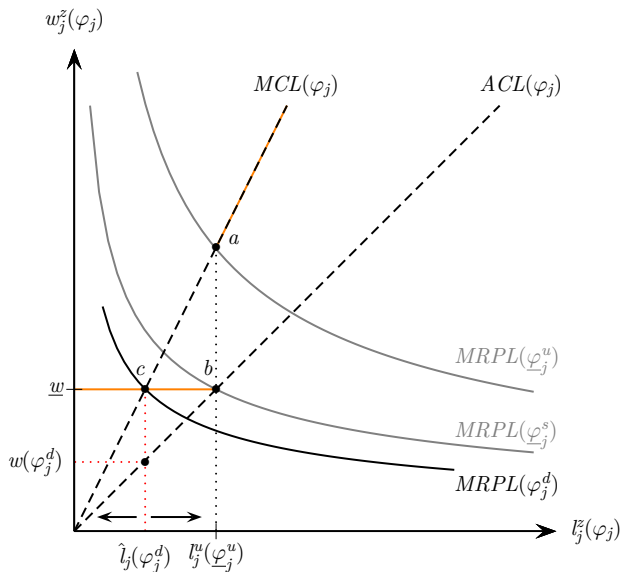
- **Unconstrained firms** are unaffected (in partial equilibrium).
- **Supply-constrained firms** *raise* employment.
- **Demand-constrained firms** may *reduce* or *raise* employment.
- **Take-away:** ΔL depends on the prevalence of firm types which is determined by the MW level *relative* to the region's average productivity (mean wage)
 \Rightarrow Reallocation between firms and regions

Supply-constrained firms



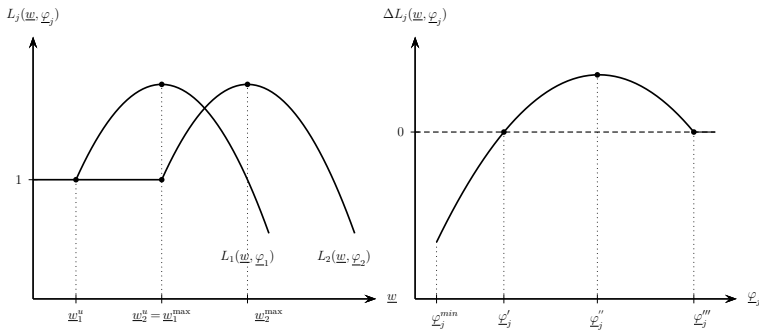
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Demand-constrained firms



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Regional employment, MW and productivity



(a) Minimum wages

(b) Regional productivity

"Hump shape" critically depends on $\varepsilon < \infty$ (monopsony)

Measuring regional employment effects

- We specify the following DGP:

$$\ln L_{j,t} = \left[\bar{f} + f(\underline{\varphi}_j) \right] I(t \geq \mathcal{J}) + \mathbf{a}_j + t\mathbf{b}_j + \epsilon_{j,t}, \quad (1)$$

- We control for local trends in employment outcomes by differencing and subtracting the pre-policy change (2012-2014; 2014-2016):

$$[\ln L_{j,t} - \ln L_{j,t-n}] - [\ln L_{j,t-n} - \ln L_{j,t-m}] = \Delta^2 \ln L_j = \bar{f} + f(\underline{\varphi}_j) + \tilde{\epsilon}_{i,t} \quad (2)$$

- $f(\underline{\varphi}_j) = \mathbb{1}(w_j^{\text{mean}} \leq \alpha_0) \times \left[\sum_{g=1}^2 \alpha_g (w_j^{\text{mean}} - \alpha_0)^g \right]$ is a g-order polynomial spline
- One-to-one mapping from $\{\alpha_0, \alpha_1, \alpha_2\}$ to $\{\underline{\varphi}'_j, \underline{\varphi}''_j, \underline{\varphi}'''_j\}$ (proxied by mean wage)
- **Akin to intensive-margin DD with treatment heterogeneity**
 - Unconstrained regions serve as reference group (absorb aggregate effects)

Hump-shape evidence



”Hump shape” provides indirect evidence of monopsony

Takeaways

- **Critical values** provide bounds for **evidence-based policy making**
 - $MW < 46\%$ counterfactual (unconstrained regions)
 - $46\% < MW \leq 64\%$ has pos. effect, but $MW > 53\%$ has neg. *marginal* effect
 - $MW > 65\%$ has negative effects
 - **Aggregate effect not identified**
 - What about shifts in labour supply from high-productivity regions?
 - And **beyond aggregate effects**, what about e.g. *welfare, local employment, commuting, aggregate labour supply, inequality, or optimal minimum wages?*
- ⇒ Take the analysis to the **spatial general equilibrium**

General equilibrium analysis

Theory: Overview

Building on the partial equilibrium framework, the GE accounts for

- Costly trade (gravity)
- Free entry of firms (endogenous number of firms)
- Free entry of workers (endogenous labour supply)
- Endogenous wages
- Endogenous prices (housing and tradables)
- Endogenous residential and workplace choice (commuting)
- Endogenous welfare

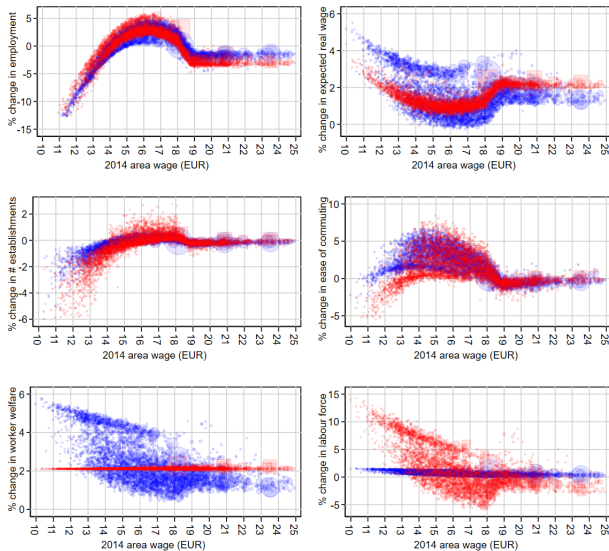
Theory

Solution

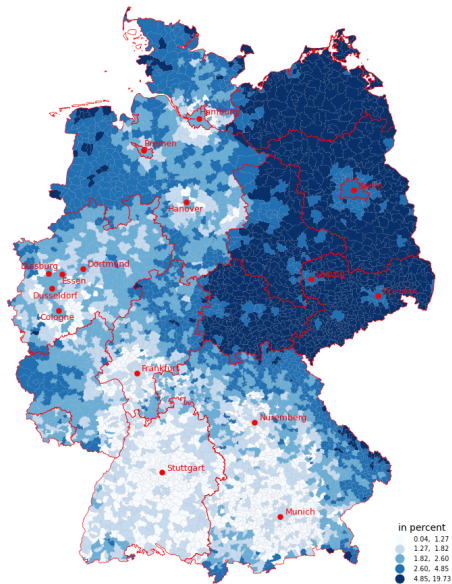
Quantification

- ① We quantify the model for 2014, prior to minimum wage.
 - ② Set, estimate, and invert structural parameters and fundamentals (data, literature, own estimation, model structure).
 - Labour supply elasticity to the firm (firm-level data, Bartik IV, conditional on municipality-year FE) $\varepsilon = 5.5$, 15% markdown \Rightarrow monopsony
 - ③ Minimum wage set relative to mean wage (numeraire).
- \Rightarrow Ready to use the model for counterfactual analysis

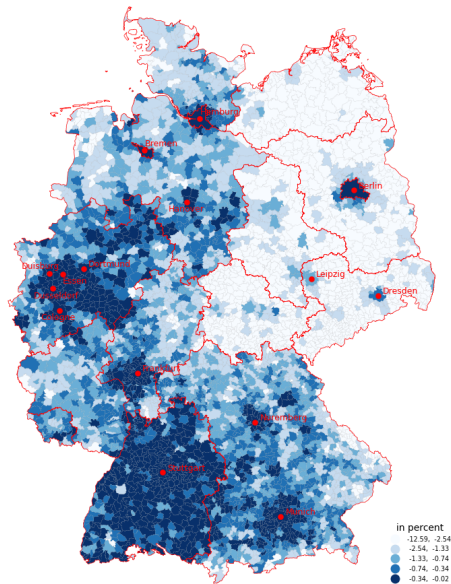
German minimum wage



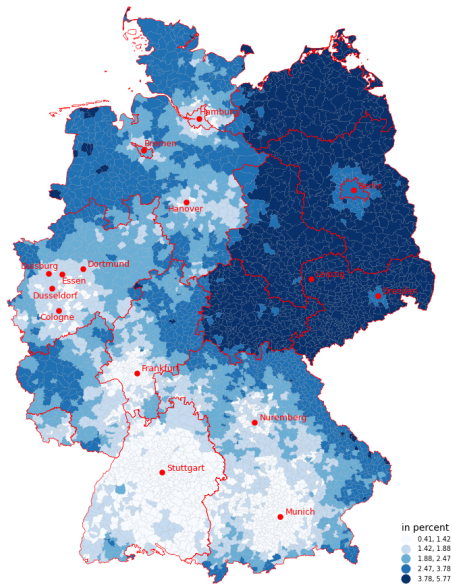
- Short run vs Long run
- **Mean effects:** Employment (-0.35%), expected real wage ($+1.6\%$), welfare ($+2.1\%$) [Table](#)
- **Regional effects:**
 - Hump-shaped employment, #firms, ease of commuting (reallocation)
 - Short-run welfare differences lead to long-run migration. [Comparison to data](#)
 - Commuting critical for hump-shape [No commuting](#)



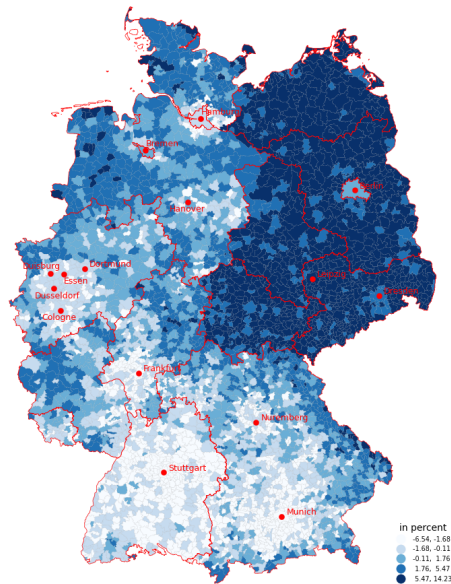
(a) Real wage, short run



(b) Employment probability, short run



(a) Welfare, short run



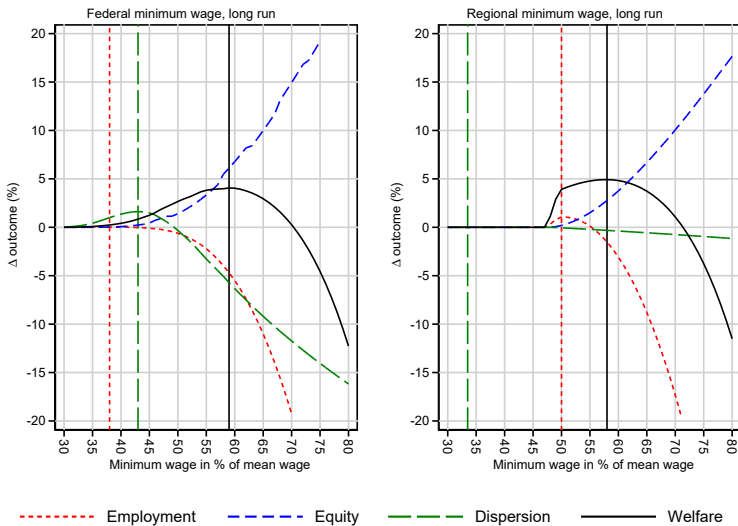
(b) Labour force, long run

Optimal minimum wage

- We derive optimal minimum wages with respect to
 - employment
 - equity (1- worker wage Gini)
 - dispersion (1- regional employment Gini)
 - welfare
- We further distinguish between
 - federal vs. regional minimum wages
 - short-run vs. long-run effects

Use *your* social welfare function to derive *your* optimal minimum wage

Optimal minimum wage schedules



Optimal minimum wage schedules

Objective	Scheme	Level rel. to		Empl.		Equity		Dispersion		Welfare	
		Mean	p50	SR	LR	SR	LR	SR	LR	SR	LR
Actual	Federal	48.0	52.8	-0.3	-0.3	1.2	1.1	0.4	0.6	2.1	2.1
Employment	Federal	38.0	41.8	0.0	0.0	0.1	0.0	0.9	1.0	0.2	0.2
Dispersion	Federal	43.0	47.3	0.0	0.0	0.3	0.2	1.4	1.6	0.8	0.8
Welfare	Federal	58.0	63.8	-3.9	-4.0	5.5	5.5	-4.9	-5.1	4.0	4.0
Employment	Regional	50.0	55.0	1.1	1.1	0.2	0.2	-0.1	-0.1	3.9	3.9
Dispersion	Regional	33.0	36.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Welfare	Regional	58.0	63.8	-1.5	-1.5	2.8	2.8	-0.3	-0.3	4.9	4.9

[Interactive map](#)

Conclusions

- Federal minimum wages are **spatially blind**, but not spatially neutral
 - moderate MW helps poor regions; high MW helps rich regions
- Regional minimum wages are **spatially neutral** because not spatially blind
 - have the potential to raise both welfare and employment

Shameless advertisement: Quantitative Spatial Economics will be offered as a PhD course in SoSe 2024 in the BSoE! Check it out if you want to learn how to solve quantitative spatial general equilibrium model clearing labour, land, and goods markets! Lots of work, but lots of fun...

Additional material

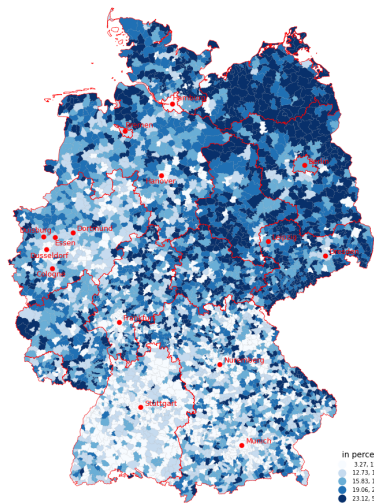
Regional minimum wage bite

- Following Machin et al. (2003):

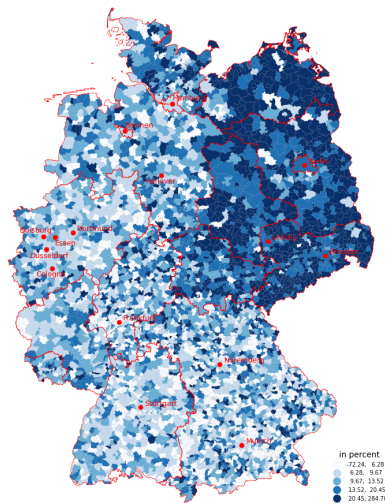
$$\mathcal{B}_i = \sum_j \frac{L_{ij}}{\sum_s L_{is}} S_j^{MW}, \quad (3)$$

- L_{ij} is the number of commuters from municipality i to j (in 2010)
 - S_j^{MW} share of workers compensated below the minimum wage in j
 - \mathcal{B}_i incorporates the bite the policy might have in j , which could transmit to i through commuting
- Analysis builds on universe (almost) of German social security and marginal employment (about 30M individuals) and 4,421 municipalities

Minimum wage bite and wage growth (POE)

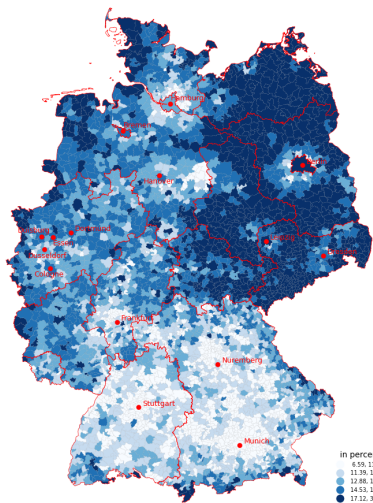


(a) Minimum wage bite in 2014

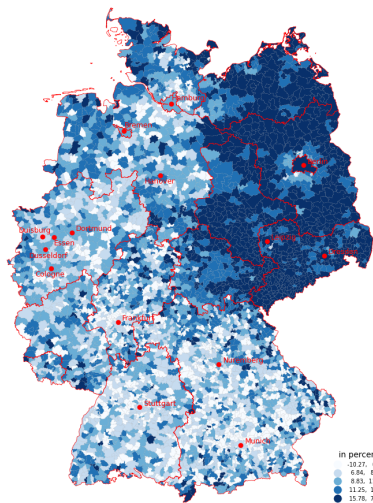


(b) 2014-2016 wage growth at 10th pct.

Minimum wage bite and wage growth (POR)

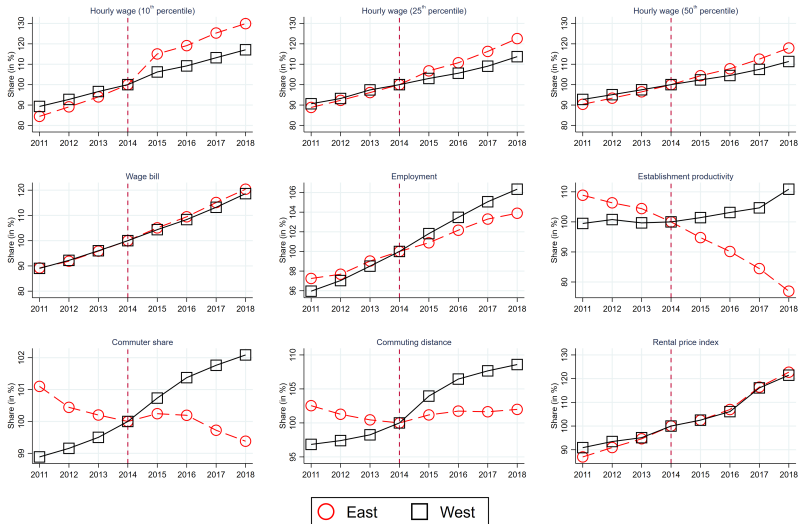


(a) Minimum wage bite in 2014

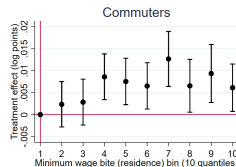
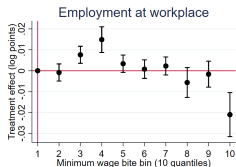
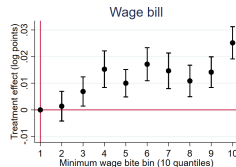
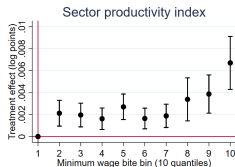
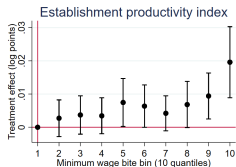
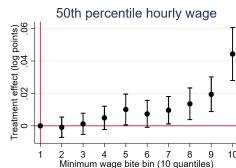
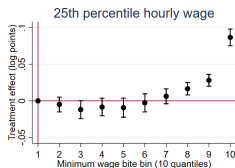
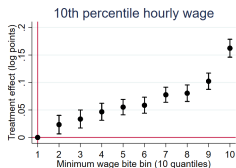


(b) 2014-2016 wage growth at 10th pct.

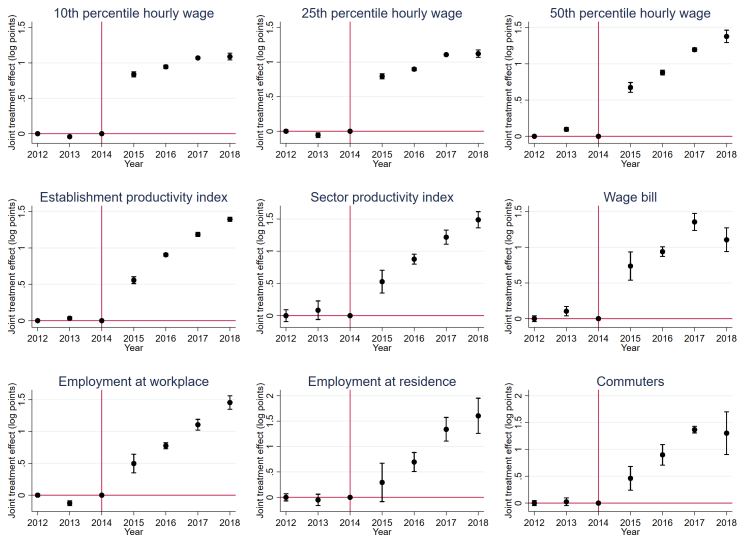
Outcome trends East vs West



Outcome trends by MWB



Outcome trends by MWB: Joint treatment effect

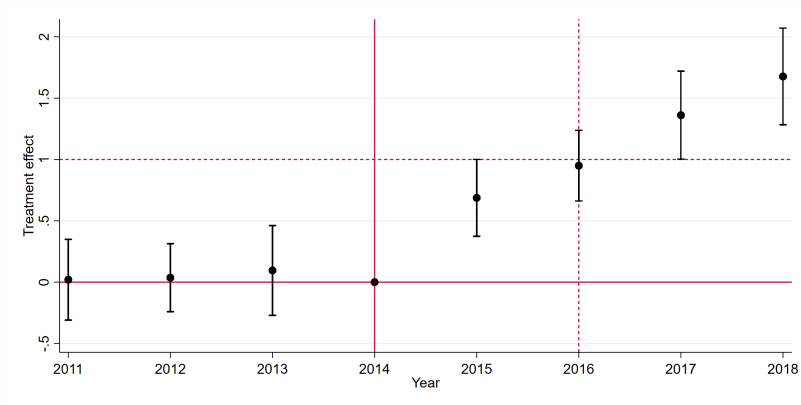


Takeaways

- Wage and employment effects
 - MW "bites", but positive aggregate employment trends continue in East and West Germany
- Reallocation effects
 - Longer commutes to more productive firms (confirms Dustmann et al., 2022)
- Prices and rents
 - Rental price index went up with some delay (e.g. Harasztosi and Lindner, 2019; Yamagishi, 2021)

Need a *general equilibrium* model with an *imperfectly competitive* labour market, *heterogeneous firms*, and *sorting* (in space and across establishments).

Dynamic DiD for hump-shape employment



⇒ Pre-trends successfully addressed

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Preferences and endowments

- The economy is composed of J locations and endowed with a working-age population \bar{N} and a fixed housing stock \bar{T} .
- Preferences of worker ν living in i and working for firm φ_j in location j are

$$U_{ij\nu}(\varphi_j) = \frac{\exp[b_{ij\nu}(\varphi_j)]}{\kappa_{ij}} \left(\frac{Q_{i\nu}}{\alpha} \right)^\alpha \left(\frac{T_{i\nu}}{1 - \alpha} \right)^{1-\alpha}, \quad (4)$$

where $Q_{i\nu}$ denotes final goods consumption, $T_{i\nu}$ residential land use, $\kappa_{ij} > 1$ commuting costs, and $\exp[b_{ij\nu}(\varphi_j)]$ is an idiosyncratic amenity shock that is Gumbel-distributed according to

$$F_{ij}[b(\varphi_{ij})] = \exp(-B_{ij} \exp\{-[\varepsilon b(\varphi_{ij}) + \Gamma'(1)]\}), \quad (5)$$

with $B_{ij} > 0$ and $\varepsilon > 0$.

Final goods index

- The goods consumption index Q_i in location i is a CES-function of a continuum of tradable varieties

$$Q_i = \left[\sum_j \int_{\varphi_j} q_{ij}(\varphi_j)^{\frac{\sigma-1}{\sigma}} d\varphi_j \right]^{\frac{\sigma}{\sigma-1}} \quad (6)$$

with $q_{ij}(\varphi_j) > 0$ denoting the quantity of variety φ_j sourced from location j and $\sigma > 1$ as the constant elasticity of substitution.

Worker welfare

- The **expected utility**, conditional on being active on the labour market, is

$$\bar{V} = \left\{ \sum_i \sum_j B_{ij} M_j \left[\frac{\Omega_j(\underline{w}) \tilde{w}_j}{\kappa_{ij} (P_i^Q)^\alpha (P_i^T)^{1-\alpha}} \right]^\varepsilon \right\}^{\frac{1}{\varepsilon}}. \quad (7)$$

where

$$\Omega_j(\underline{w}) \equiv \frac{\eta \Phi_j^W(\underline{w})^{\frac{1}{\varepsilon}} \Phi_j^L(\underline{w})}{\Phi_j^R(\underline{w}) - (1 - \eta) \Phi_j^\Pi(\underline{w})} \quad (8)$$

is a composite adjustment factor that captures various channels through which the minimum wage affects the wage index, \tilde{w}_j denotes the average wage, P_i^Q is the final-goods price index and M_j counts the number of firms (varieties).

Labour market entry

- We can express the **labour force participation rate** as

$$\mu = \frac{\bar{V}^\zeta}{\bar{V}^\zeta + A}, \quad (9)$$

where ζ is the Gumbel shape parameter that is a transformation of the Hicksian extensive-margin labour supply elasticity, and A is the shift parameter that captures the leisure amenity.

- The Gumbel distribution of idiosyncratic taste shocks implies that **expected welfare across all workers** (working, searching, and abstaining) takes the following form:

$$\bar{V} = \left(A + \bar{V}^\zeta \right)^{\frac{1}{\zeta}} \quad (10)$$

Model solution

- The general equilibrium of the model can be referenced by the following vector of seven variables $\{\tilde{w}_i, \tilde{v}_i, M_j, P_i^T, L_i, N_i, P_i^Q\}_{i=1}^J$ and the scalars $\{\mu, \bar{V}\}$.
- Given the equilibrium values of these variables and scalars, all other endogenous objects can be determined conditional on the model's primitives.
- This equilibrium vector solves the following seven sets of equations:
 - ① Income equals expenditure
 - ② Average residential income
 - ③ Firm entry
 - ④ Housing market clearing
 - ⑤ Aggregate local employment
 - ⑥ Residential choice ($N_i = \lambda_i^N N$)
 - ⑦ Price index
- The conditions needed to determine the scalars $\{\mu, \bar{V}\}$ are
 - ① Labour force participation
 - ② Labour market clearing

Quantification I

- We need to uncover structural parameters $\{\underline{w}, k, \alpha, \sigma, \epsilon, \zeta, \mu\}$ and the structural fundamentals $\{\tau_{ij}, \kappa_{ij}, B_{ij}, \underline{\varphi}_j, \bar{T}_i, f_j^e, A\}$.
- We quantify the model using data from 2014 (year before MW introduction).
- We borrow $\{\alpha, \zeta\}$ from the literature and set σ such that all parameter restrictions of the model are satisfied.
- We infer all other primitives from the data using observed values of $\{P^T, \lambda_{ij}N_i, M_j, w_j(\omega), \tilde{w}_j, (p_{ij}q_{ij}), \mu\}$.

Quantification II

- ① **Expenditure share on housing** ($1 - \alpha$). We set $1 - \alpha = 0.33$ (Ahlfeldt and Pietrostefani, 2019, Federal Statistical Office, 2020).
- ② **Labour force participation rate** (μ). $\mu = 73.6\%$ (2014) as reported by the German Federal Statistical Office.
- ③ **Working-age population** (\bar{N}). Based on IAB employment data, $\bar{N} = N/\mu$.
- ④ **Reservation utility heterogeneity** (ζ). Combining Hicksian extensive-margin labour supply elasticity $\tilde{\zeta} = 2$ and $\mu = 0.739$ delivers $\zeta = 0.8$.
- ⑤ **Preference heterogeneity** (ε). Theory-consistent estimate of ε from an establishment-level regression of the log of wage against the log of employment (Bartik IV), controlling for area fixed effects. This yields $\varepsilon = 5.2$.

Quantification III

⑥ **Productivity heterogeneity and elasticity of substitution (k, σ).**

Intuitively, we identify k by fitting a Pareto cumulative distribution function (CDF) of wages as conventional in the trade literature. To meet parameter constraints, we nest the estimation of k using a GMM estimator into a grid search over σ values. We choose $\sigma = 1.5$ (closest to the conventions in the literature) and obtain an estimate for k of 0.53.

⑦ **Minimum wage (\underline{w}).** Defined relative to the numeraire (worker-weighted mean wage).

⑧ **Trade cost (τ_{ij}).** Estimated from gravity equation of bilateral trade flows between counties within Germany allowing for inner-German border effect and origin-specific distance effects.

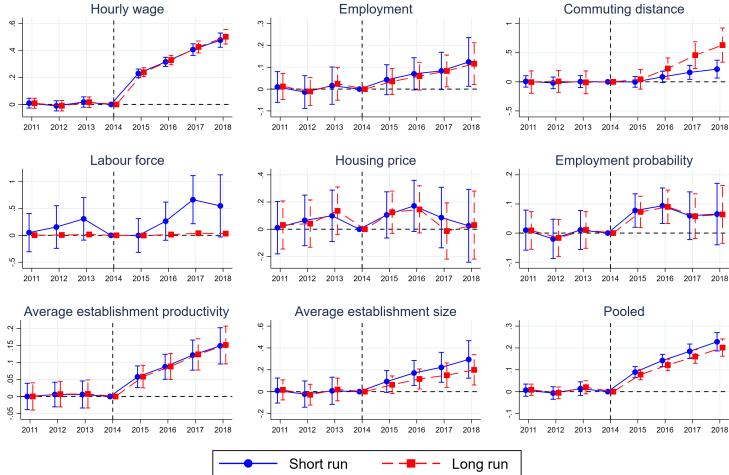
Quantification IV

- ⑨ **Fundamental productivity ($\underline{\varphi}_j$).** Given values of $\{L_j, N_i, \lambda_{ij|i}^N, \tilde{w}_j, M_j, \varepsilon, \sigma, \tau_{ij}\}$, we can invert $\underline{\varphi}_j$ from “Income = Expenditure”.
- ⑩ **Ease of commuting ($B_{ij}\kappa_{ij}^{-\varepsilon}$).** Given values of $\{\alpha, \varepsilon, \sigma, k, \tau_{ij}, \underline{\varphi}, \lambda_{ij|i}^N, M_j, \tilde{w}_j, P_i^T\}$, we invert $B_{ij}\kappa_{ij}^{-\varepsilon}$ using the unconditional commuting probabilities and a conventional fixed-point solver.
- ⑪ **Start-up space (f_j^e).** Given values of $\{\varepsilon, \sigma, M_j, P_j^T, \tilde{w}_j, L_j\}$, obtain f_j^e from the free-entry condition.
- ⑫ **Housing supply (\bar{T}_i).** For given values of $\{\lambda_{ij|i}^N, \tilde{w}_i, L_i\}$, we use the proportionality feature between profits and wage income in the housing market clearing condition.
- ⑬ **Leisure amenity (A).** Using values of $\{\mu_i, M_j, \tilde{w}_j, P_i^T, \underline{\varphi}_j, \tau_{ij}, B_{ij}\kappa_{ij}^{-\varepsilon}\}^{\alpha, \varepsilon, \sigma, \zeta, k}$, we invert fundamental utility A using equations for the price index, aggregate worker welfare and the labour force participation rate. [Back](#)

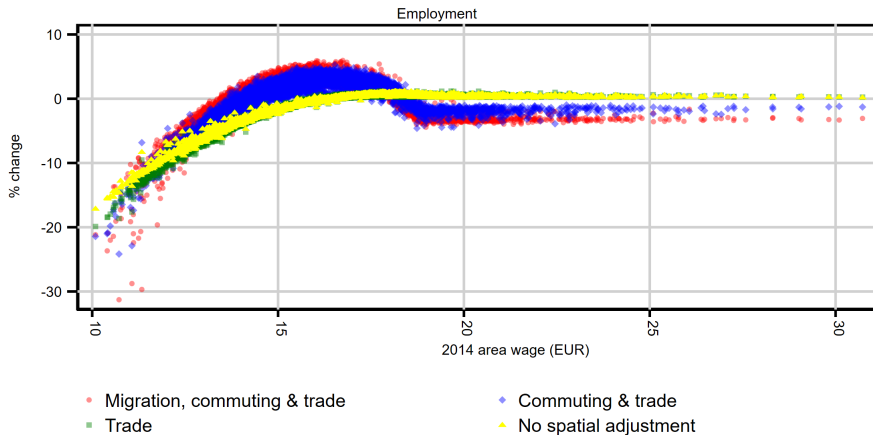
Effects of German MW

	Short run			Long run		
	Mean	Min	Max	Mean	Min	Max
<i>Panel a: Employment</i>						
Employment at workplace (L)	-0.250	-21.31	5.350	-0.350	-25.91	5.810
Labour supply at residence (N)	0.590	0.120	1.550	0.590	-6.540	14.23
Employment probability (L/H)	-0.820	-19.99	0	-0.880	-21.15	0
<i>Panel b: Wage and prices</i>						
(Normalized) wage (\tilde{w})	0.320	-1.360	25.72	0.390	-1.310	24.70
Real tradables price index (P^Q)	-3.040	-4.620	-2.200	-2.930	-5.630	-1.600
Real housing rent (P^T)	-1.040	-7.170	1.100	-1.070	-5.390	2.520
<i>Panel c: Welfare components</i>						
Exp. real wage $\tilde{v} [(P^Q)^\alpha (P^T)^{(1-\alpha)}]$	1.620	-0.260	5.510	1.630	0.370	4.350
# establishments (M)	-0.100	-7.290	0.920	-0.120	-16.43	2.770
Ease of commuting ($B\kappa^{-\epsilon}$)	1.160	-4.290	7.090	0.880	-14.04	8.440
<i>Panel d: Welfare</i>						
Worker welfare working (V)	2.910	0.560	7.830	2.860	2.860	2.860
Worker welfare, all (\mathcal{V})	2.150	0.410	5.770	2.110	2.110	2.110

Model vs. data



No commuting



- Without migration and commuting, there would be no hump shape!